

Solutions to Maths workbook - 1 | Quadratic Equations

Level – 3

Daily Tutorial Sheet – 15

- 168.** The AM of $x+3, x-1$ is $\frac{x+3+x-1}{2}$, i.e., $x+1$
 Put $x+1 = y$. Then $x+3 = y+2, x-1 = y-2$.
 $\therefore$ the equation becomes $(y+2)^5 - (y-2)^5 \geq 244$
 or $y^5 + {}^5C_1 \cdot y^4 \cdot 2 + {}^5C_2 \cdot y^3 \cdot 2^2 + {}^5C_3 \cdot y^2 \cdot 2^3 + {}^5C_4 \cdot y \cdot 2^4 + {}^5C_5 \cdot 2^5$
 $-(y^5 - {}^5C_1 \cdot y^4 \cdot 2 + {}^5C_2 \cdot y^3 \cdot 2^2 - {}^5C_3 \cdot y^2 \cdot 2^3 + {}^5C_4 \cdot y \cdot 2^4 - {}^5C_5 \cdot 2^5) \geq 244$
 or $2[{}^5C_1 \cdot y^4 \cdot 2 + {}^5C_3 \cdot y^2 \cdot 2^3 + {}^5C_5 \cdot 2^5] \geq 244$ or $10y^4 + 80y^2 + 32 \geq 122$
 or $y^4 + 8y^2 - 9 \geq 0$ or $(y^2 + 9)(y^2 - 1) \geq 0$
 $\Rightarrow y^2 - 1 \geq 0$ because $y^2 + 9 > 0$ for real y .
 $\therefore y^2 - 1 \geq 0$ holds if $y \leq -1$ or $y \geq 1$
 Now, $y \leq -1 \Rightarrow x+1 \leq -1$ or $x \leq -2$
 $y \geq 1 \Rightarrow x+1 \geq 1$ or $x \geq 0$
 Hence, $x \leq -2$ or $x \geq 0$
 Therefore, the solution set $= (-\infty, -2] \cup [0, +\infty)$.
- 169.** Find $x > 0, y > 0$ satisfying the system of equations $x^{x+y} = y^n$ and $y^{x+y} = x^{2n} \cdot y^n$ where $n > 0$.
 Let $x^{x+y} = y^n$ (1)
 $y^{x+y} = x^{2n} \cdot y^n$ (2)
 Multiplying (1) and (2), $(xy)^{x+y} = x^{2n} \cdot y^{2n} = (xy)^{2n}$
 $\therefore x+y = 2n$ (provided $xy \neq 1$) or $xy = 1$.
 If $xy = 1$, (1) becomes $x^{\frac{1}{x}} = x^{-n}$
 $\Rightarrow x = 1$ or $x + \frac{1}{x} = -n$
 If $x = 1$ then $y = \frac{1}{x} = 1$
 If $x + \frac{1}{x} = -n < 0$ then $x > 0$ cannot hold.
 If $xy \neq 1, x+y = 2n$; then (1) gives $x^{2n} = y^n$
 $\Rightarrow \left(\frac{x^2}{y}\right)^n = 1 \Rightarrow n = 0$ or $x^2 = y$.
 But $n > 0$; So $x^2 = y$.
 $\therefore x+y = 2n$ gives $x+x^2 = 2n$ or $x^2 + x - 2n = 0$ or $x = \frac{-1 \pm \sqrt{1+8n}}{2}$
 But $x > 0$; So $x = \frac{-1 + \sqrt{1+8n}}{2}$ and $y = 2n - x = 2n - \frac{-1 + \sqrt{1+8n}}{2}$

$$\therefore \text{ solutions are } x = 1, y = 1 \quad \text{and} \quad x = \frac{-1 + \sqrt{1+8n}}{2}, y = \frac{1 + 4n - \sqrt{1+8n}}{2}.$$

170. Here, $x = \{x\} + [x]$ according to the question.

Clearly, x is an integer or x is not an integer.

If x is an integer, let $x = n$. Then $[x] = n, \{x\} = 0$.

$$\therefore \text{ The equation } 4\{x\} = x + [x] \quad \dots(1)$$

$$\text{Gives } 0 = n + n; \quad \therefore \quad n = 0; \quad \therefore \quad x = 0.$$

If x is positive and not an integer, let $x = n + k$ where n is a positive integer and $0 < k < 1$

$$\therefore \quad [x] = n, \{x\} = k.$$

$$\text{So, from (1), } 4k = n + k + n \quad \text{or} \quad 3k = 2n; \quad \therefore \quad k = \frac{2n}{3}.$$

$$\text{But } 0 < k < 1; \quad \therefore \quad 0 < \frac{2n}{3} < 1 \quad \text{or} \quad 0 < n < \frac{3}{2};$$

$$\therefore \quad n = 1 \text{ and So, } k = \frac{2}{3} \quad \therefore \quad x = n + k = 1 + \frac{2}{3} = \frac{5}{3}.$$

If x is negative and not an integer, let $x = -n - k$, where n is any positive integer and $0 < k < 1$.

$$\therefore \quad [x] = -n, \{x\} = -k. \text{ So, from (1), } 4(-k) = -n - k + (-n)$$

$$\text{or} \quad -4k = -2n - k \quad \text{or} \quad 3k = 2n; \quad \therefore \quad k = \frac{2n}{3}.$$

$$\text{But } 0 < k < 1; \quad \therefore \quad 0 < \frac{2n}{3} < 1 \quad \text{or} \quad 0 < 2n < 3; \quad \therefore \quad 0 < n < \frac{3}{2}.$$

$$\text{So } n = 1 \quad \therefore \quad x = -n - k = -1 - \frac{2}{3} = -\frac{5}{3}.$$

$$\text{Hence, the solutions are } x = 0, \pm \frac{5}{3}.$$

171. Here, $\log_{|x+6|^2} 2 \times \log_2 (x^2 - x - 2) = 1 \quad \text{or} \quad \log_{|x+6|^2} (x^2 - x - 2) = 1$

$$\{\therefore \log_n m \times \log_m p = \log_n p\}$$

$$\Rightarrow \quad x^2 - x - 2 = |x + 6|^2 \quad \dots(1)$$

Clearly, for logarithmic functions to be defined

$$|x + 6|^2 > 0; |x + 6|^2 \neq 1; x^2 - x - 2 > 0$$

$$\Rightarrow \quad x \neq -6; |x + 6| \neq 1; \quad x^2 - x - 2 > 0$$

$$\Rightarrow \quad x \neq -6; \quad x \neq -5, -7; \quad x < -1 \quad \text{or} \quad x > 2 \quad \dots(2)$$

$$\text{Now, (1)} \quad \Rightarrow \quad x^2 - x - 2 = (x + 6)^2 \quad \{\therefore |x|^2 = x^2\}$$

$$\text{or} \quad x^2 - x - 2 = x^2 + 12x + 36 \quad \text{or} \quad 13x = -38; \quad \therefore \quad x = \frac{-38}{13}$$

$$\text{Clearly, } x = \frac{-38}{13} \text{ satisfies all the conditions of (2).}$$

$$\text{Hence, the solution is } x = \frac{-38}{13}.$$

172. Here $2a |x - a| = x^2 - 3a^2 \quad \text{or} \quad |x - a| = \frac{2}{2a}(x^2 - 3a^2) \quad \text{or} \quad |x - a| = \frac{1}{2a}(x^2 - 3a^2)$

But $|x - a|$ is positive and a is negative.

$$\text{So, } x^2 - 3a^2 \leq 0$$

$$\therefore x - a = \pm \frac{1}{2a}(x^2 - 3a^2), \text{ provided } x^2 - 3a^2 \leq 0.$$

$$\text{Now, } x - a = \frac{1}{2a}(x^2 - 3a^2) \Rightarrow 2ax - 2a^2 = x^2 - 3a^2$$

$$\text{or } x^2 - 2ax - a^2 = 0 \quad \text{or } x = \frac{2a \pm \sqrt{4a^2 + 4a^2}}{2} = (1 \pm \sqrt{2})a$$

$$\text{Again, } x - a = -\frac{1}{2a}(x^2 - 3a^2) \Rightarrow 2ax - 2a^2 = -x^2 + 3a^2$$

$$\text{or } x^2 + 2ax - 5a^2 = 0 \quad \therefore x = \frac{-2a \pm \sqrt{4a^2 + 20a^2}}{2} \quad \text{or } x = \frac{-2a \pm 2\sqrt{6}a}{2} = (-1 \pm \sqrt{6})a$$

$$\therefore \text{ We get } x = (1 + \sqrt{2})a, (1 - \sqrt{2})a, (-1 + \sqrt{6})a, (-1 - \sqrt{6})a.$$

$$\text{When } x = (1 + \sqrt{2})a,$$

$$x^2 - 3a^2 = (1 + \sqrt{2})^2 a^2 - 3a^2 = 2\sqrt{2}a^2 > 0.$$

$$\text{When } x = (1 - \sqrt{2})a,$$

$$x^2 - 3a^2 = (1 - \sqrt{2})^2 a^2 - 3a^2 = -2\sqrt{2}a^2 < 0$$

$$\text{When } x = (-1 + \sqrt{6})a$$

$$x^2 - 3a^2 = (-1 + \sqrt{6})^2 a^2 - 3a^2 = 4a^2 - 2\sqrt{6}a^2 < 0$$

$$\text{When } x = (-1 - \sqrt{6})a$$

$$x^2 - 3a^2 = (-1 - \sqrt{6})^2 a^2 - 3a^2 = 4a^2 + 2\sqrt{6}a^2 > 0$$

$$\text{As } x^2 - 3a^2 \leq 0, \text{ the solutions are } (1 - \sqrt{2})a, (-1 + \sqrt{6})a.$$

173. Here $(2^{2x^2})^2 \leq 3.2x^2 \cdot 2^x + 4.2^{2x}$

$$\text{or } (2^{x^2})^2 \leq 3.2x^2 \cdot 2^x + 4 \cdot (2^x)^2 \quad \text{or } (2^{x^2})^2 - 3.2x^2 \cdot 2^x - 4 \cdot (2^x)^2 \leq 0$$

$$\text{or } (2^{x^2} - 4.2^x)(2^{x^2} + 2^x) \leq 0 \quad \dots (1)$$

$$\text{For all real } x, 2^{x^2} + 2^x > 0. \text{ So (1) will hold if } 2^{x^2} - 4.2^x \leq 0$$

$$\text{or } 2^{x^2} \leq 2^{x+2} \quad \text{or } x^2 \leq x+2, \text{ because the base } 2 > 1$$

$$\therefore x^2 - x - 2 \leq 0 \quad \Rightarrow \quad -1 \leq x \leq 2$$

$$\text{Hence the required solution-set } = [-1, 2].$$